Exercise 12

For matrix A in the previous problem, find a nonzero $\mathbf{x} \in \mathbb{R}^3$ such that $A\mathbf{x} = \mathbf{0}$.

Solution

In Problem 11

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 3 & 3 \end{bmatrix}$$

The aim is to find a three-dimensional vector,

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix},$$

such that

$$A\mathbf{x} = \mathbf{0}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This matrix equation implies the following system of equations.

$$\left.\begin{aligned} x + 2y + 3z &= 0 \\ y + z &= 0 \\ 3y + 3z &= 0 \end{aligned}\right\}$$

Notice that the second and third equations are equivalent. Solve either of them for y

$$y = -z$$

and plug it into the first equation.

$$x + 2(-z) + 3z = 0$$

Solve for x.

$$x = -z$$

As a result, any three-dimensional vector of the form

$$\mathbf{x} = \begin{bmatrix} -z \\ -z \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

will work, where z is an arbitrary constant (a free variable). For example, choosing $z = 1$ gives

$$\mathbf{x} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}.$$